

MCA
(SEM I) THEORY EXAMINATION 2018-19
DISCRETE MATHEMATICS

Time: 3 Hours

Total Marks: 70

Note: 1. Attempt all Sections. If require any missing data; then choose suitably.

SECTION A

1. Attempt *all* questions in brief. 2 x 7 = 14

- a. Define the Power set .
If $A = \{1,2,3\}$ find $P(A)$ and $n\{P(A)\}$.
- b. Define the Cartesian Product of sets.
If $U = \{1,2,3,4,5,6,7,8\}$, $A = \{2,4,6,8\}$, and $B = \{3,5,6,7\}$ then find $A \times B$, $A - B$?
- c. Define Complemented lattice example.
- d. Find the dual of the Boolean : $f = x'yz' + x'y'z$.
- e. Consider the Poset $S = (\{1, 2, 3, 4, 6, 9, 12, 18, 36\}, /)$.
Find the Greatest Lower Bound and Least Upper Bound of the sets $\{6,18\}$ and $\{4,6,9\}$.
- f. Define the term Tautology , and Contradiction .
Show that $(p \rightarrow (q \wedge r)) \rightarrow (\sim r \rightarrow \sim p)$ is a tautology.
- g. State the "Pigeonhole Principle".

SECTION B

2. Attempt any *three* of the following: 7 x 3 = 21

- a. Define the Composite relation. And Let set $A = \{1,2,3\}$, $B = \{p,q,r\}$, $C = \{x,y,z\}$ and the relations are, $R = \{(1,p), (1,r), (2,q), (3,q)\}$ and $S = \{(p,y), (q,x), (r,z)\}$, then compute RoS .
- b. Let D_m denote the positive divisors of integers m ordered by divisibility.
Draw the Hasse diagrams of : a) D_{24} , b) D_{15}
- c. Convert the following Boolean Function in DNF as well as CNF :
 $f(x,y,z) = xy' + xz + xy$.
- d. Define the terms converse , contrapositive , and inverse of a proposition .
Show that $(p \rightarrow q) \wedge (r \rightarrow q) \equiv (p \vee r) \rightarrow q$
- e. Everybody in a room shakes hands with everybody else. The total number of handshakes is 66. How many people are there in the room?

SECTION C

- 3. Attempt any *one* part of the following:** **7 x 1 = 7**
- (a) Show that for any two sets A and B in set theory: $A - (A \cap B) = A - B$.
- (b) State the Principle of Mathematical Induction. And show that $8^n - 3^n$ is divisible by 5 for $n \geq 1$.
- 4. Attempt any *one* part of the following:** **7 x 1 = 7**
- (a) Let $A = \{1, 2, 3, 6\}$ and Let \leq the divisibility relation on A and let $B = \{\phi, \{a\}, \{b\}, \{a, b\}\}$ and the relation \subseteq be the relation \subseteq . Then show that (A, \leq) and (B, \subseteq) are isomorphic posets.
- (b) If $A = \{1, 2, 3, 4, 6, 12, 18, 36\}$ be ordered by the relation "a divides b". Then draw the Hasse diagram.
- 5. Attempt any *one* part of the following:** **7 x 1 = 7**
- (a) Draw Karnaugh map (K-map) and simplify the following Boolean function:

$$f(x, y, z, w) = \sum(0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 11)$$
- (b) State the De-Morgan's Laws of Boolean Algebra. And Express the following Boolean function in Sum of minterms and Product of maxterm:

$$f(x, y, z) = x + y'z$$
- 6. Attempt any *one* part of the following:** **7 x 1 = 7**
- (a) Define the term Arguments.
 Prove the validity of the following argument "If I get the job and work hard, then I will get promoted. If I get promoted, then I will be happy. I will not be happy. Therefore, either I will not get the job or I will not work hard".
- (b) Construct the truth table $((p \Rightarrow q) \vee (q \Rightarrow p)) \Leftrightarrow p$.
 Is the proposition: Tautology, Contradiction or Contingency?
- 7. Attempt any *one* part of the following:** **7 x 1 = 7**
- (a) Determine S^2a and $S^{-2}a$ for the following numeric functions:

$$a_r = \begin{cases} 2^r & , 0 \leq r \leq 3 \\ 2^{-r} & , r \geq 4 \end{cases}$$
- (b) Solve the following recurrence relation:

$$a_n + 6a_{n-1} + 9a_{n-2} = 3$$
. Given that $a_0 = 0$ and $a_1 = 1$